Front-door Versus Back-door Adjustment with Unmeasured Confounding*

Adam N. Glynn and Konstantin Kashin, *Harvard University*

aglynn@fas.harvard.edu, kkashin@fas.harvard.edu

Contributions

- Provide formulas for the large sample bias of front-door estimators (Pearl 1995) for both ATE and ATT with general patterns of measured and unmeasured confounding and multiple mediators
- Formulas agnostic about whether mediator causal effects are well-defined
- Bias from the front-door approach can be compared to the VanderWeele and Arah (2011) bias formulas for standard back-door covariate adjustments (e.g., matching adjustments for ATT)
- Front-door approaches will be preferred to back-door approaches in many applications
- In some applications with one-sided non-compliance, control units will be unnecessary

Front-door for ATT

- $Y(a_1)$ is potential outcome under active treatment and $Y(a_0)$ is potential outcome under control
- Observed covariates X and unobserved covariates U allow identification of ATT
- M is a set of measured mediators

$$\begin{aligned}\text{ATT} &= \text{E}[Y|a_1] - \text{E}[Y(a_0)|a_1] \\ &= \text{E}[Y|a_1] - \sum_x \sum_u \text{E}[Y|a_0, x, u] \cdot P(u|x, a_1) \cdot P(x|a_1)\end{aligned}$$

$$\text{Front-door adjustment} = \text{E}[Y|a_1] - \sum_x \sum_m P(m|a_0, x) \cdot \text{E}[Y|a_1, m, x] \cdot P(x|a_1)$$

Programs with One-Sided Non-Compliance

- Let a_1 denote self-selection into program and a_0 denote opt-out
- $M = 1$ is program participation, $M = 0$ is no-show
- One-sided non-compliance: $P(M = 1|a_0, x) = 0$ and $P(M = 0|a_0, x) = 1$

$$\text{Front-door adjustment} = \text{E}[Y|a_1] - \underbrace{\sum_x \text{E}[Y|a_1, M = 0, x] \cdot P(x|a_1)}_{\text{Treated non-compliers}}$$

$$\text{Back-door adjustment} = \text{E}[Y|a_1] - \underbrace{\sum_x \text{E}[Y|a_0, x] \cdot P(x|a_1)}_{\text{Controls}}$$

Simplified Bias Comparison

Assumption (1) Relationships don't vary across strata of X
Assumption (2) U is binary

Under (1) and (2), general back-door bias formulas simplify (VanderWeele and Arah 2011):

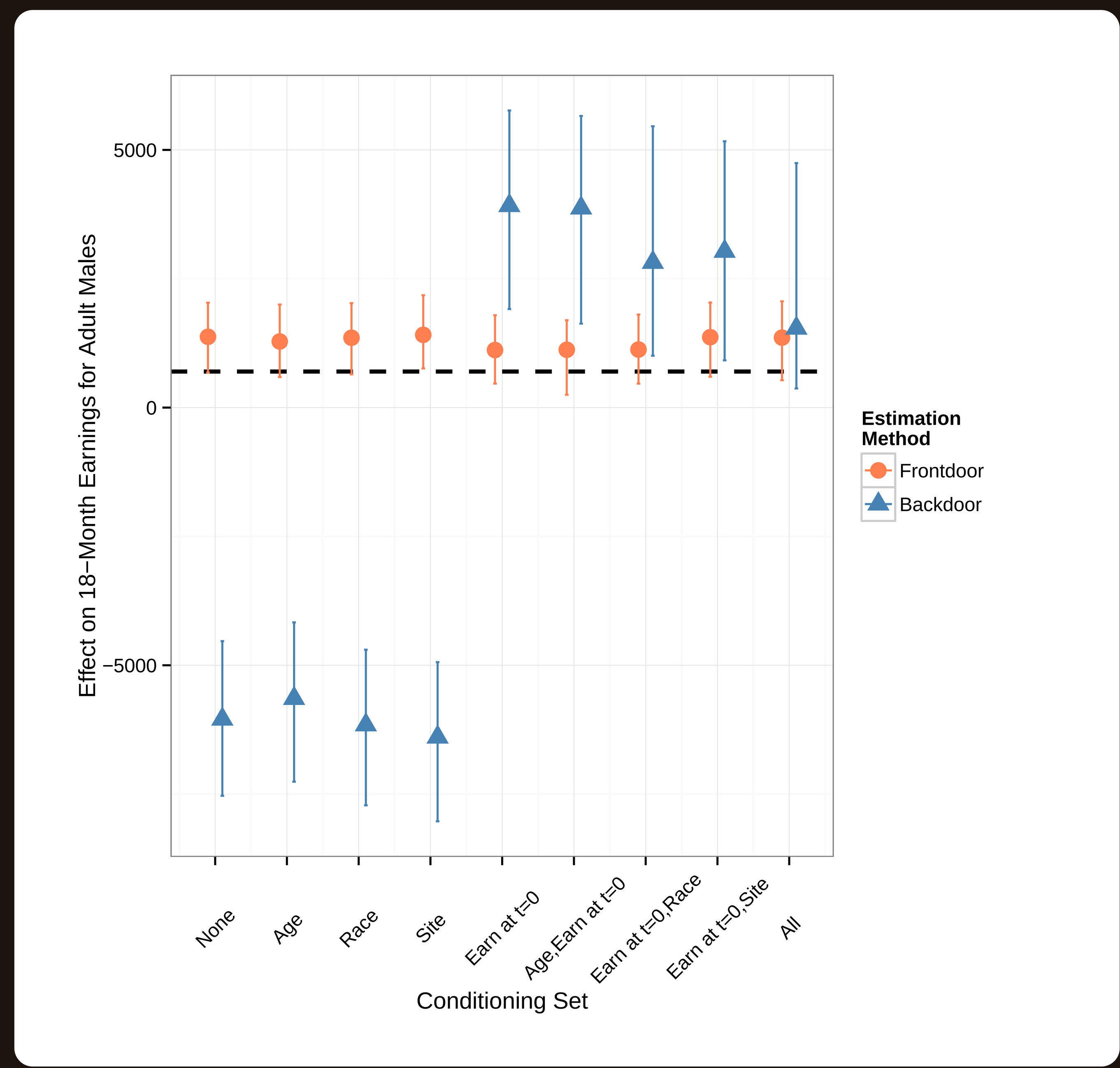
$$\text{Back-door Bias} = \underbrace{(\text{E}[Y|U = 1, a_0, x] - \text{E}[Y|U = 0, a_0, x])}_{\text{Direct "effect" of } U} \cdot \underbrace{[P(U = 1|a_1, x) - P(U = 1|a_0, x)]}_{\text{Back-door imbalance}}$$

Under (1) and (2), general front-door bias formulas simplify:

$$\text{Front-door Bias} = \underbrace{(\text{E}[Y|U = 1, a_0, x] - \text{E}[Y|U = 0, a_0, x])}_{\text{Direct "effect" of } U} \cdot \underbrace{[P(U = 1|a_1, x) - P(U = 1|a_1, x, M = 0)]}_{\text{Front-door imbalance}}$$

$$- \underbrace{[\sum_u P(u|a_1, M = 0, x) \cdot (\text{E}[Y|u, a_1, M = 0, x] - \text{E}[Y|u, a_0, M = 0, x])]}_{\text{Direct "effect" of } A}$$

Job Training Partnership Act Results



Two interesting cases for sensitivity analysis:

- Direct "effect" of U, back-door imbalance, and front-door imbalance are all non-negative and direct "effect" of A is non-positive
- Direct "effect" of A is ≈ 0 and |front-door imbalance| < |back-door imbalance|

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