Front-door Versus Back-door Adjustment with Unmeasured Confounding: Bias Formulas for Front-door and Hybrid Adjustments[1]

Adam Glynn and Konstantin Kashin
Harvard University

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[1] Presented at the 2013 Joint Statistical Meetings, Montreal, Quebec, Canada.

Outline

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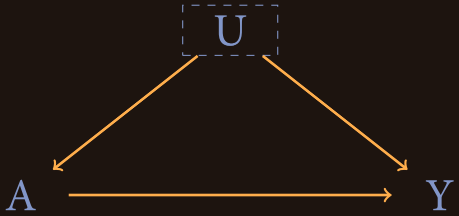
Causal Effects with Unmeasured Confounding



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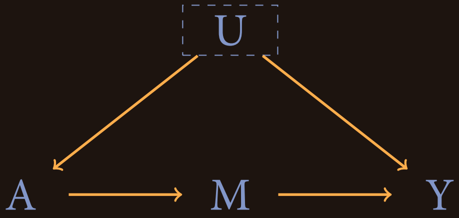

We can use post-treatment variable M to identify causal effects (Pearl, 1995).

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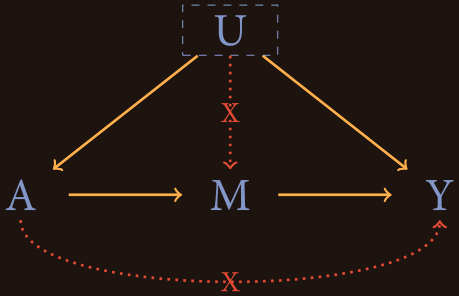

Pearl's (1995) front-door criterion enables point-identification of causal effect.

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Still relatively little use of the front-door technique and extensions.

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- In some applications with one-sided noncompliance, control units will be unnecessary

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Motivation

Illustrative Example: ATT with One-Sided Noncompliance

Application: JTPA

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$$\begin{aligned} \mathrm{ATT} &= \mathrm{E}[Y|a_1] - \mathrm{E}[Y(a_0)|a_1] \\ &= \mathrm{E}[Y|a_1] - \sum_x \sum_u E[Y|a_0, x, u] \cdot P(u|x, a_1) \cdot P(x|a_1) \end{aligned}$$

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$$- \underbrace{\left[\sum_u P(u|a_1,M=0,x) \cdot \left(E[Y|u,a_1,M=0,x] - E[Y|u,a_0,M=0,x]\right)\right]}_{\text{Direct "effect" of } A}$$

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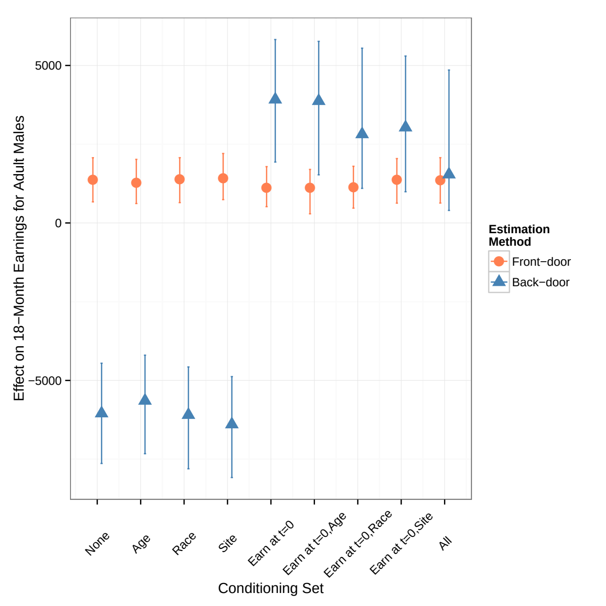
- Measure program sign-up impact as ATT on 18-month earnings post-randomization
- One-sided noncompliance: people who didn't sign-up not allowed to receive JTPA services and some sign-ups drop out

Front- & Back-door Estimates for Adult Males



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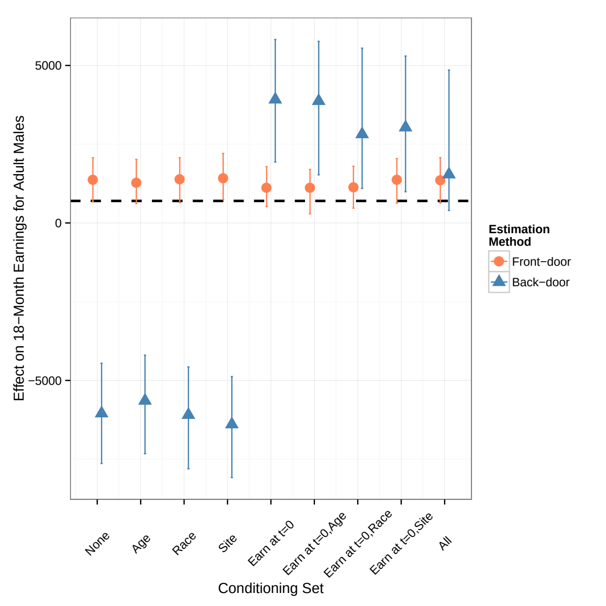

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- Findings have surprising implications for research design
    - For non-randomized programs, it may be more important to collect compliance information on the treated units than outcome information on the control units